Характеристики и характеристич-я форма системы ур-й одномерной газовой динамики

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial v}{\partial x} + v\frac{\partial \rho}{\partial x} = 0; \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0; \quad \frac{\partial S}{\partial t} + v\frac{\partial S}{\partial x} = 0$$

$$p = p(\rho, S); \quad \frac{\partial p}{\partial x} = \underbrace{\left(\frac{\partial p}{\partial \rho}\right)}_{= c^2}\frac{\partial \rho}{\partial x} + \underbrace{\left(\frac{\partial p}{\partial S}\right)}_{\sigma}\frac{\partial S}{\partial x}; \quad \vec{u} = \begin{pmatrix} \rho \\ v \\ S \end{pmatrix}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \rho}{\partial t} + \rho\dfrac{\partial v}{\partial x} + v\dfrac{\partial \rho}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} + \dfrac{c^2}{\rho}\dfrac{\partial \rho}{\partial x} + \dfrac{\sigma}{\rho}\dfrac{\partial S}{\partial x} = 0 \\ \dfrac{\partial S}{\partial t} + v\dfrac{\partial S}{\partial x} = 0 \end{cases} \qquad A = \begin{bmatrix} v & \rho & 0 \\ \frac{c^2}{\rho} & v & \frac{\sigma}{\rho} \\ 0 & 0 & v \end{bmatrix}$$

$$\frac{\partial \vec{u}}{\partial t} + A\frac{\partial \vec{u}}{\partial x} = 0; \quad \det|A - \lambda E| = 0$$

$$\begin{vmatrix} v-\lambda & \rho & 0 \\ \frac{c^2}{\rho} & v-\lambda & \frac{\sigma}{\rho} \\ 0 & 0 & v-\lambda \end{vmatrix} = (v-\lambda)\left((v-\lambda)^2 - c^2\right) = 0;$$

$\lambda_0 = v,\ \lambda_+ = v+c,\ \lambda_- = v-c;\ (v-\lambda) = \pm c$

$lA = \lambda l;$

$$(l_1, l_2, l_3)\begin{pmatrix} v & \rho & v \\ \frac{c^2}{\rho} & v & \frac{\sigma}{\rho} \\ 0 & 0 & v \end{pmatrix} = \lambda(l_1, l_2, l_3)$$

$$l_1 v + l_2\frac{c^2}{\rho} = \lambda l_1 \Rightarrow l_1(v-\lambda) + l_2\frac{c^2}{\rho} = 0$$

$$l_1\rho + l_2 v = \lambda l_2 \Rightarrow l_1\rho = l_2(\lambda - v); \quad l_2\frac{\sigma}{\rho} + l_3 v = \lambda l_3 \Rightarrow l_2\frac{\sigma}{\rho} = l_3(\lambda - v)$$

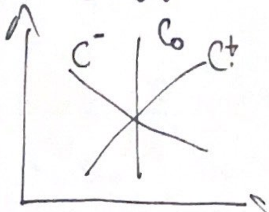
① $\lambda_+ = v + c \qquad l_1(-c) = -l_2\frac{c^2}{\rho};\ l_1\rho = l_2 c;\ l_2\frac{\sigma}{\rho} = l_3 c$

$$l^{(1)} = \begin{pmatrix} c \\ \rho \\ \frac{\sigma}{c} \end{pmatrix}; \quad \lambda^+ = v + c; \quad l^{(2)} = \begin{pmatrix} c \\ \rho \\ \frac{\sigma}{c} \end{pmatrix}; \quad \lambda^- = v - c; \quad l^{(2)} = \begin{pmatrix} -c \\ \rho \\ -\frac{\sigma}{c} \end{pmatrix}$$

$$\lambda^0 = v; \quad l^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad i = 1, 2, 3 \qquad \sum_{i=1}^{3} l_i^k\left(\frac{\partial u_i}{\partial t} + \lambda_k\frac{\partial u_i}{\partial x}\right) = 0$$

$k=1$:

$$\begin{cases} c\left(\frac{\partial \rho}{\partial t}+(v+c)\frac{\partial \rho}{\partial x}\right)+\rho\left(\frac{\partial v}{\partial t}+(v+c)\frac{\partial v}{\partial x}\right)+\frac{\sigma}{c}\left(\frac{\partial S}{\partial t}+(v+c)\frac{\partial S}{\partial x}\right)=0 \\ -c\left(\frac{\partial \rho}{\partial t}+(v-c)\frac{\partial \rho}{\partial x}\right)+\rho\left(\frac{\partial v}{\partial t}+(v-c)\frac{\partial v}{\partial t}\right)-\frac{\sigma}{c}\left(\frac{\partial S}{\partial t}+(v-c)\frac{\partial S}{\partial x}\right)=0 \\ \frac{\partial S}{\partial t}+v\frac{\partial S}{\partial x}=0 \end{cases}$$



Инвар-ты Римана для системы ур-й одномерной газовой динамики

$$\begin{cases} \frac{\partial v}{\partial t}+\frac{c}{\rho}\frac{\partial \rho}{\partial t}+(v+c)\left(\frac{\partial v}{\partial x}+\frac{c}{\rho}\frac{\partial \rho}{\partial x}\right)+\frac{\sigma}{\rho c}\left(\frac{\partial S}{\partial t}+(v+c)\frac{\partial S}{\partial x}\right)=0 \\ \frac{\partial v}{\partial t}-\frac{c}{\rho}\frac{\partial \rho}{\partial t}+(v-c)\left(\frac{\partial v}{\partial x}-\frac{c}{\rho}\frac{\partial \rho}{\partial x}\right)-\frac{\sigma}{\rho c}\left(\frac{\partial S}{\partial t}+(v-c)\frac{\partial S}{\partial x}\right)=0 \\ \frac{\partial S}{\partial t}+v\frac{\partial S}{\partial x}=0 \end{cases}$$

Рассм. $S=S_0(x)$ (изэнтроп-ий случай)

$$\frac{\partial S}{\partial x}=0,\ \frac{\partial S}{\partial t}=0 \Rightarrow \begin{cases} \left(\frac{\partial v}{\partial t}+\frac{c}{\rho}\frac{\partial \rho}{\partial t}\right)+(v+c)\left(\frac{\partial v}{\partial x}+\frac{c}{\rho}\frac{\partial \rho}{\partial x}\right)=0 \\ \left(\frac{\partial v}{\partial t}-\frac{c}{\rho}\frac{\partial \rho}{\partial t}\right)+(v-c)\left(\frac{\partial v}{\partial x}-\frac{c}{\rho}\frac{\partial \rho}{\partial x}\right)=0 \end{cases}$$

$$c^2=\left.\frac{\partial p}{\partial \rho}\right|_{S_0};\ p(\rho,S_0);\ c(\rho)$$

$$\frac{c(\rho)}{\rho}\cdot\frac{\partial \rho}{\partial x}=\frac{\partial}{\partial x}\varphi(\rho);\ \varphi(\rho)=\int_{\rho_0}^{\rho}\frac{c(\xi)}{\xi}d\xi;\ \begin{cases} \frac{\partial}{\partial t}(v+\varphi(\rho))+(v+c)\frac{\partial}{\partial x}(v+\varphi(\rho))=0 \\ \frac{\partial}{\partial t}(v-\varphi(\rho))+(v-c)\frac{\partial}{\partial x}(v-\varphi(\rho))=0 \end{cases}$$

$$\tau^+=v+\varphi(\rho);\ \tau^-=v-\varphi(\rho)$$

$$\frac{\partial}{\partial t}(\tau^+)+(v+c)\frac{\partial}{\partial x}\tau^+=0$$

$$\frac{\partial}{\partial t}(\tau^-)+(v-c)\frac{\partial}{\partial x}\tau^-=0$$

C^+: $\frac{dx}{dt}=v+c$ и $\tau^+=const$ на C^+

C^-: $\frac{dx}{dt}=v-c$ и $\tau^-=const$ на C^-

$$v=\frac{\tau^++\tau^-}{2};\ \varphi(\rho)=\frac{\tau^+-\tau^-}{2};\ \varphi(\rho)=\int_{\rho_0}^{\rho}\frac{c(\xi)}{\xi}d\xi;\ \rho=\varphi^{-1}\left(\frac{\tau^+-\tau^-}{2}\right)$$

$$c(\rho)=c\left(\varphi^{-1}\left(\frac{\tau^+-\tau^-}{2}\right)\right)=\psi\left(\frac{\tau^+-\tau^-}{2}\right)$$

$$\begin{cases} \frac{\partial}{\partial t}\tau^++\left(\frac{\tau^++\tau^-}{2}+\psi\left(\frac{\tau^+-\tau^-}{2}\right)\right)\frac{\partial \tau^+}{\partial x}=0 \\ \frac{\partial}{\partial t}\tau^-+\left(\frac{\tau^++\tau^-}{2}-\psi\left(\frac{\tau^+-\tau^-}{2}\right)\right)\frac{\partial \tau^-}{\partial x}=0 \end{cases}$$